

## Chapter 6

### IMPROVEMENTS TO ENERGY BALANCE MODEL

In Chapter 5, estimates of evaporation from the energy balance model (EBM) were shown to be less well correlated with actual evaporation than was the quantity  $(T_{o,max} - T_{d,max})$ . Since the EBM was:

$$\int E \, dt = S(T_{o,max} - T_{d,max}) / L_e \quad [6-1]$$

where

$$S = 6[1 + (2/\pi)^{0.5}] (\rho C_p / r + 4\epsilon \sigma \bar{T}_m^3) \quad [6-2]$$

it became apparent that some of the assumptions made in the development of the EBM, and implicit in the term  $S$ , were suspect.

This chapter concerns EBM improvements that come from elimination of, or changes in, the original assumptions. First, integration on a smaller time step, then better estimates of dry and drying soil temperatures, and finally, improvement in the estimation of sensible heat flux are introduced. At each step, the improved EBM is described in equation form and its performance documented. The chapter closes with a discussion of energy flux terms that were neglected in the model and suggestions for further work.

### Numerical Integration.

Re-arranging Equation 5-20, the energy balance model was:

$$\int E \, dt = \int [\rho C_p / r + 4\epsilon \sigma \bar{T}_m^3] (T_o - T_d) / L_e \, dt \quad [6-3]$$

where the instantaneous temperature depression ( $T_o - T_d$ ) was given by Equation 5-21. Numerical integration of Equation 6-3, on a finer time step than 12 hours, e.g. 1/2 hour, would allow closer interaction between wind speed (and thus  $r$ ) and the quantity ( $T_o - T_d$ ) as both factors change in magnitude over the day. The finer time step would also allow the quantity  $\bar{T}_m^3$  to vary over the day in accordance with the values of  $T_o$  and  $T_d$  thus reducing the error associated with the assumption that  $\bar{T}_m^3$  was constant. Therefore,  $\bar{T}_m$  was redefined:

$$\bar{T}_m = (T_o + T_d) / 2 \quad [6-4]$$

A computer program for the numerical integration calculated the evaporation,  $E$ , for each microlysimeter on a half-hourly basis and summed it for the period desired (in this case either from -3 to 9 hours or from 7 to 7 hours on the next day). Sunrise was taken as zero hours. The daily totals of evaporation,  $E_{est}$ , in mm were regressed on the actual evaporation,  $E_a$ , in mm. In writing the computer code, assumptions 1, 2, 3, 4, 5 and 6 were taken as true. Wind speed was assumed constant only over one-half hour periods.

When the period of summation was -3 to 9 hours, assumptions 10 and 11 were taken as true; but these assumptions were dropped when the period of summation was taken as 0 to 24 hours.

The program was compared to the original EBM by setting  $\bar{T}_m$  constant (assumption 8) and by letting  $(T_o - T_d)$  be described by Equation 5-21 (assumption 7). Taking -3 to 9 hours as the period of interest, the numerically estimated evaporation,  $E_{est}$  (mm), was regressed against actual evaporation, with dummy variables in the model for the treatments. The resulting  $R^2$  value of 0.553 was only slightly larger than the 0.546 gotten from regressing the predictions of Equation 5-26 against actual evaporation. Again, the model overestimated evaporation by about 100 percent. Letting  $\bar{T}_m = (T_o + T_d)/2$  resulted in only a very slight improvement in  $R^2$  value to 0.556. The value of  $\bar{T}_m$  was defined by Equation 6-4 in subsequent trials.

Since there was no reason to think that evaporation did not occur during the period from 9 to -3 hours, a second set of estimates was calculated using the period from 7 to 7 hours (i.e. the 24 hour period from time of weighing on one day to the time of weighing on the next day). Regression of  $E_a$  versus  $E_{est}$  with dummy variables for the treatments resulted in only a slightly higher  $R^2$  value of 0.572. Since the time of weighing on the first day after irrigation varied greatly, another set of estimates of  $E$  was calculated with the

numerical integration starting at the time of first weighing for each ML. The unexpected result was an  $R^2$  value of 0.549 for regression of  $E_a$  versus  $E_{est}$  with dummy variables for the treatments, a lower  $R^2$  value than for the uncorrected predictions (Table 6-1). Again, this model overestimated evaporation by almost 100 percent on average.

These last three results were surprising. Integrating on a half-hourly basis should have allowed for a much better interaction between wind speed, which was highly variable over the day, and soil temperature. The expected result was a much better fit between actual and estimated  $E$  but the actual increase in  $R^2$  was minor. Likewise, correction for time of weighing on the first day should have resulted in much better prediction since as much as 50 percent of total evaporation took place on the first day and the weighings took place between 9:20 AM and 3:15 PM. However this correction resulted in a lower, not higher,  $R^2$ . Since correction for time of weighing resulted in a worse fit, it became reasonable to question the adequacy of the sine wave approximation for surface temperature since this was the only assumption which was time dependent.

**Table 6-1.**

Regression analyses for daily evaporation,  $E_a$ , (mm) with the estimated evaporation,  $E_{est}$ , (mm) from numerical integration of Equation 5-23 as the independent variable; and dummy variables for length and wall type treatments. Numerical integration was from 7 AM to 7 AM and was begun at the time of first weighing for each microlysimeter.

Model:  $E_a = b_0 + b_1 E_{est}$

$r^2 = 0.508$ ,  $n = 136$ .

parameter	estimate	std. error	significance
intercept	-0.739	0.193	0.000
$E_{est}$	0.562	0.048	0.000

Model:  $E_a = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 + b_5 x_5 + b_6 E_{est}$   
 $+ b_{16} x_{16} + b_{26} x_{26} + b_{36} x_{36} + b_{46} x_{46} + b_{56} x_{56}$

See Appendix E for explanation of model.

$r^2 = 0.549$ ,  $n = 136$

parameter	estimate	std. error	significance
intercept	-0.422	0.525	0.423
$x_1$	0.076	0.756	0.921
$x_2$	0.203	0.669	0.762
$x_3$	-1.148	0.730	0.119
$x_4$	-0.412	0.655	0.531
$x_5$	-1.323	0.816	0.107
$E_{est}$	0.506	0.128	0.000
$x_{16}$	-0.086	0.185	0.642
$x_{26}$	-0.044	0.179	0.806
$x_{36}$	0.197	0.173	0.256
$x_{46}$	0.121	0.164	0.461
$x_{56}$	0.256	0.184	0.166

Equations:

$E_a = -0.347 + 0.420 E_{est}$ , 10 cm, steel  
 $E_a = -0.230 + 0.462 E_{est}$ , 10 cm, plastic  
 $E_a = -1.570 + 0.704 E_{est}$ , 20 cm, steel  
 $E_a = -0.834 + 0.628 E_{est}$ , 20 cm, plastic  
 $E_a = -1.746 + 0.762 E_{est}$ , 30 cm, steel  
 $E_a = -0.422 + 0.506 E_{est}$ , 30 cm, plastic

### Temperature Depression.

In the previous section it was shown that integration of the EBM, with a half-hour time step and starting with the time of weighing of ML's on the first day, resulted in a worse fit between  $E_a$  and  $E_{est}$  than integration with no correction for time of weighing. Since the temperature depression,  $(T_o - T_d)$ , as given by Equation 5-21 included the only time dependent assumptions, it appeared that this sine wave approximation might cause the inaccuracy observed when correcting for time of weighing. Examination of Equation 5-20 reveals that over-estimation of the temperature depression term could also cause the observed over-estimation of evaporation. The temperature depression,  $(T_o - T_d)$ , was represented in Equation 5-20 by

$$T_o - T_d = 0.5(T_{o,max} - T_{d,max})(1 + \sin(wt)) \quad [6-5]$$

The assumptions made in deriving Equation 6-5 were that soil surface temperature was described by a sine function, that the minimum and maximum temperatures occurred simultaneously in both dry and drying soils, and that the minimum temperatures were equal in both dry and drying soils. Thus Equation 6-5 predicts that the temperature depression will always be positive whereas it was shown in Chapter 3 to be usually negative at night.

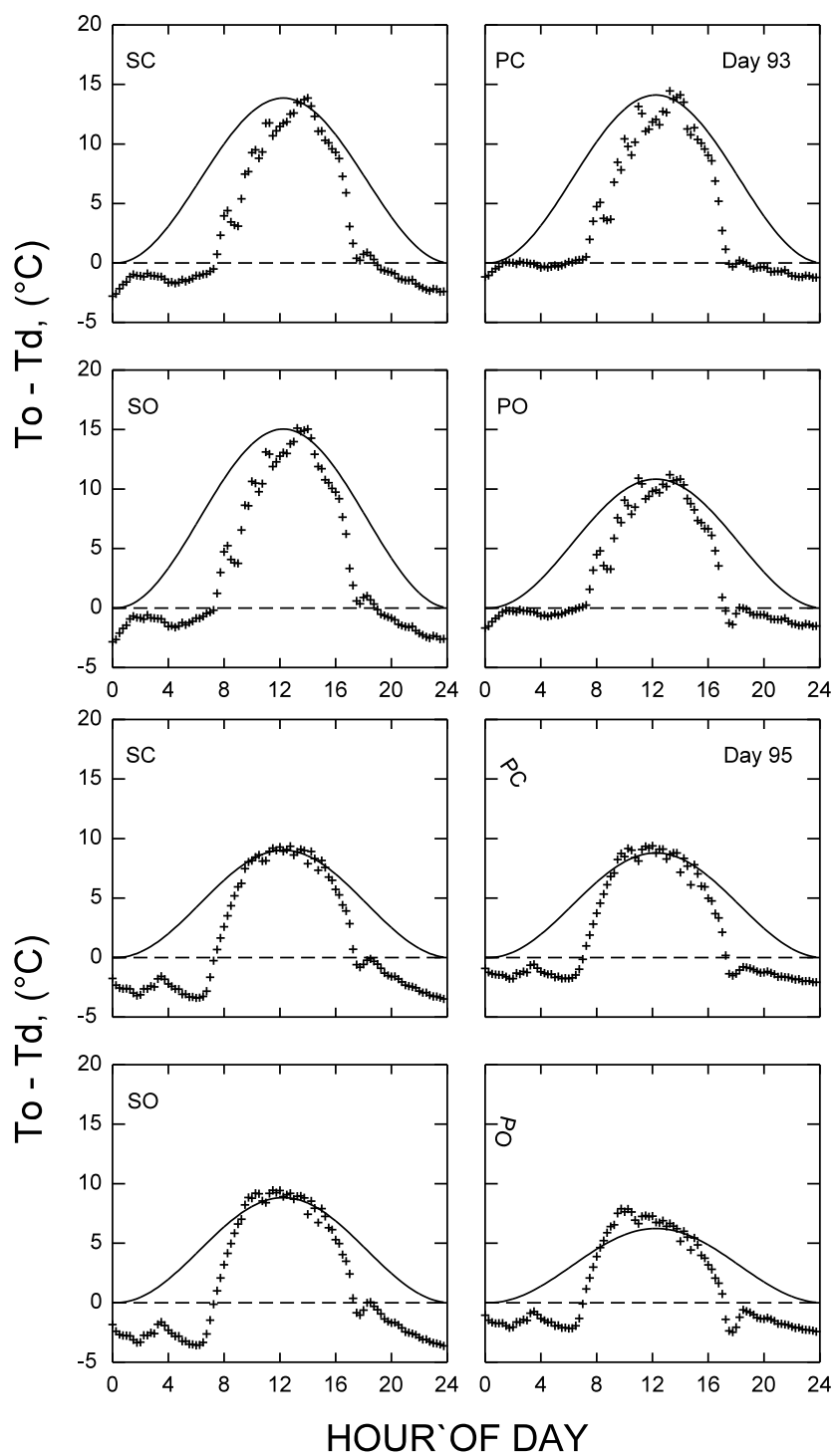
For field measurements with the IR thermometer it was assumed that the minimum temperatures occurred just before

dawn (about 6 AM) and that the maximums occurred at about 1 PM. One obvious problem with these assumptions is that the time between minimum and maximum in the field is only about 7 hours whereas the half-period of the sine function is 12 hours. Another problem is that the sine wave does not approximate the surface temperature well (Figures 3-5 through 3-8).

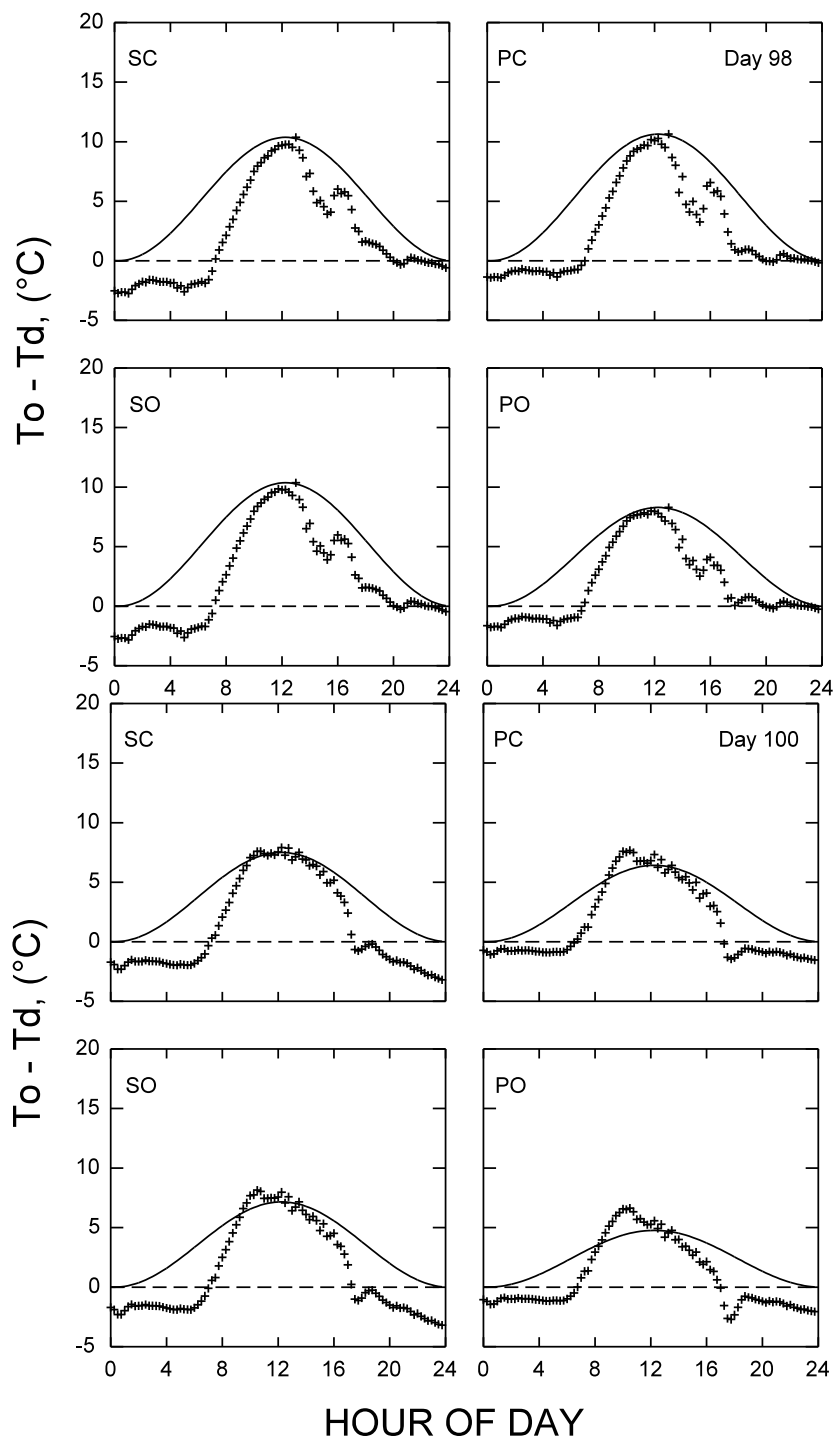
The actual temperature depression, as measured with thermistors in the field, was plotted versus the temperature depression predicted by Equation 6-5, on a 15 minute basis, (Figures 6-1 and 6-2). Data were from 2 replicates of 4 treatments of 30 cm ML's: steel walls with either closed or open bottoms and plastic walls with closed or open bottoms. The difference between actual and predicted ( $T_o - T_d$ ) was large and could easily account for the over-prediction of evaporation by numerical integration of Equation 5-23. Also, since actual ( $T_o - T_d$ ) was negative for about 12 hours of the day, when the sine function estimated it to be positive, it became clear why correction for time of first weighing of the ML's caused inaccurate estimation of evaporation (see previous section). For half of each diurnal period the relationship, between the temperature depression and the sensible heat flux term in the EBM, was signed positive when it was really negative.

Several features of these graphs are interesting. Only at about 12:00 were the actual and predicted ( $T_o - T_d$ ) values nearly equal, at other times actual and predicted values differed considerably, especially at sunrise and sunset when the difference could amount to 10 °C. At night, actual ( $T_o - T_d$ ) values were almost always negative, more so for steel than for plastic ML's. Negative values were probably due to the fact that the drying soils had considerably higher thermal conductivity than the dry soil and so could conduct heat from subsurface layers to the surface more readily. Thus drying soil surfaces would be warmer than the dry soil at night when the major skyward energy flux component is probably longwave radiation emitted by the soil surface.

The fact that steel ML's exhibited more negative ( $T_o - T_d$ ) values at night reflects the much higher thermal conductivity of steel compared to plastic. Daytime maximum values of ( $T_o - T_d$ ) were nearly equal for both treatments of steel ML's and for plastic ML's with closed bottoms. Plastic ML's with open bottoms exhibited lower maximum ( $T_o - T_d$ ) values on all days. This behavior is probably due to drainage of water from the open-bottomed plastic ML's resulting in quicker drying and in higher daytime surface temperatures compared to plastic ML's with closed bottoms. Steel ML's exhibited lower daytime



**Figure 6-1.** Comparison of actual  $(T_o - T_d)$  with that predicted by the sine wave approximation of Equation 6-5. Days 93 and 95.



**Figure 6-2.** Comparison of actual  $(T_o - T_d)$  with that predicted by the sine wave approximation of Equation 6-5. Days 98 and 100.

temperatures irrespective of whether or not their bottoms were closed, probably because heat conduction by the metal walls was large enough to overshadow any differences.

Comparison of the graphs of wind speed (Figure 3-4) with those of temperature depression shows a slight dependence of  $(T_o - T_d)$  on wind speed. On day 95, just before 4 AM, a several fold increase in wind speed is associated with a decrease in the absolute value of  $(T_o - T_d)$ . On day 98, at about 3 PM, a large increase in wind speed is again associated with a marked decrease in the absolute value of  $(T_o - T_d)$ . However, in both cases the decrease in  $|T_o - T_d|$  could have been associated with increasing cloud cover.

Comparisons were made of daily evaporation, as estimated using actual  $(T_o - T_d)$ , to evaporation estimated using  $(T_o - T_d)$  values calculated using Equation 6-5. For the latter estimates Equation 5-23 was numerically integrated as before but using the average maximum and minimum soil surface temperatures,  $T_{o,max}$  and  $T_{d,max}$ , as measured by thermistor for each ML treatment (2 replicates). For the former calculations Equation 6-3 was integrated numerically using actual  $(T_o - T_d)$  values measured at half hour intervals. The integration was from 7 AM to 7 AM on the next day, days 92 and 94 being omitted due to lack of data.

**Table 6-2.**

Estimated evaporation, mm, using  $(T_o - T_d)$  calculated using the sine function [sine], and using actual  $(T_o - T_d)$  [actual]. Averaged temperatures from two ML's for each treatment.

	Treatments							
	SC		SO		PC		PO	
	sine	actual	sine	actual	sine	actual	sine	actual
Day								
93	6.70	2.91	7.27	3.27	6.82	3.42	5.25	2.57
95	3.80	1.55	3.72	1.53	3.70	1.90	2.63	1.19
96	3.74	1.51	3.61	1.52	3.44	1.97	2.58	1.28
97	4.23	1.63	4.22	1.72	4.03	2.02	3.08	1.42
98	5.75	2.69	5.75	2.81	5.90	3.07	4.60	2.35
99	4.10	1.69	3.94	1.74	3.81	2.01	2.83	1.40
100	3.46	1.26	3.31	1.22	2.96	1.54	2.21	0.83
Sum	31.78	13.24	31.82	13.81	30.66	15.93	23.18	11.04
Ave.	4.54	1.89	4.55	1.97	4.38	2.28	3.31	1.58

Percentage increase in estimated E when estimated  $(T_o - T_d)$  is used instead of actual  $(T_o - T_d)$ :

130.2	122.3	99.4	104.3
145.2	143.1	94.7	121.0
147.7	137.5	74.6	101.6
159.5	145.4	99.5	116.9
113.8	104.6	92.2	95.7
142.6	126.4	89.6	102.1
174.6	171.3	92.2	166.3
Average 140 %, steel		Average 104 %, plastic	

Treatments:

SC = steel ML with covered bottom.

SO = steel ML with open bottom.

PC = plastic ML with covered bottom.

PO = plastic ML with open bottom.

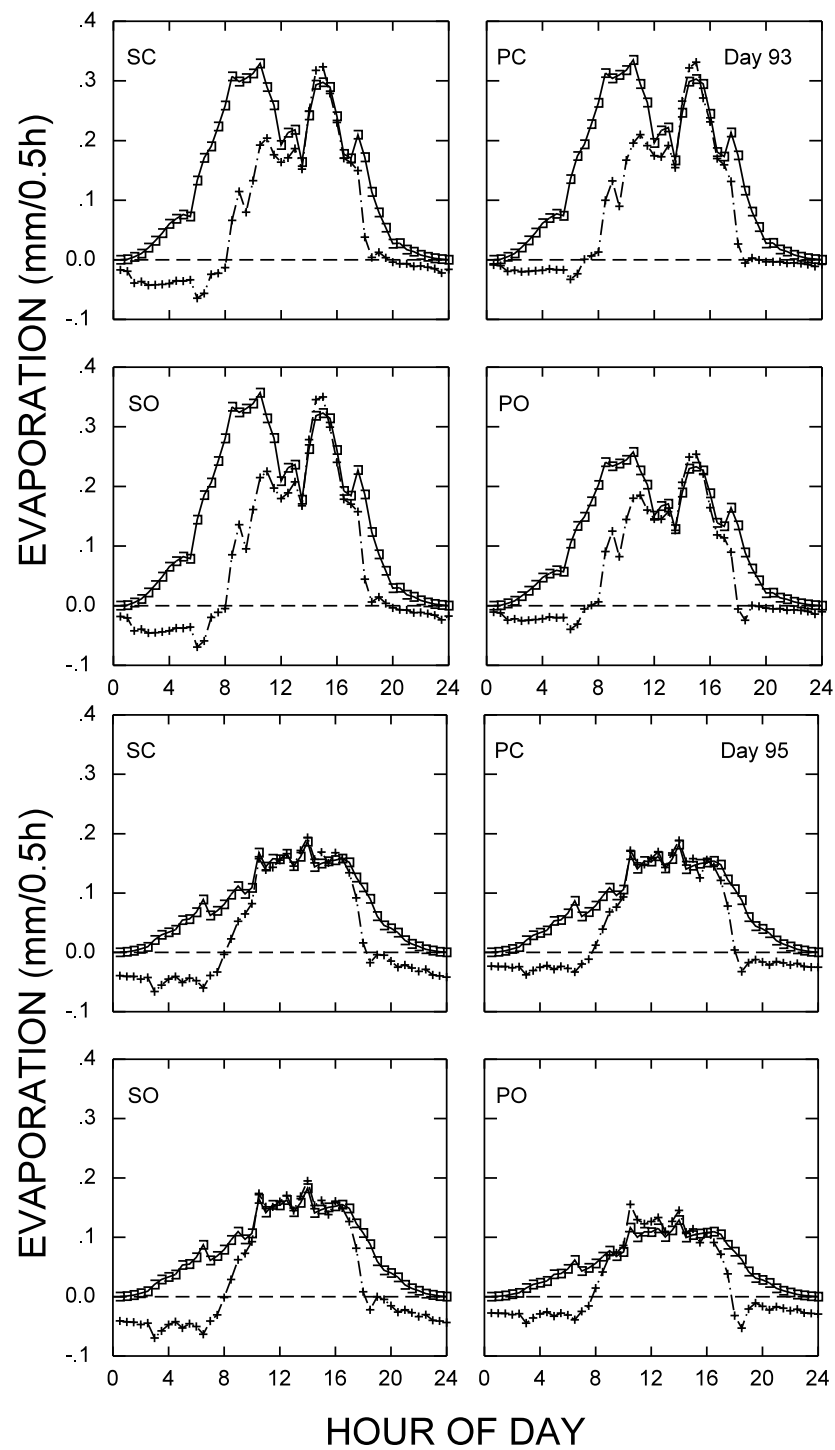
Estimates of daily  $E$ , using the sine function and Equation 5-23 to calculate  $(T_o - T_d)$ , were from 75 to 175 percent greater than estimates of  $E$  made using actual  $(T_o - T_d)$  values (Table 6-2, Figures 6-3 and 6-4). The discrepancy was greatest for steel ML's since they exhibited the most negative  $(T_o - T_d)$  values at night. Steel ML's with closed bottoms acted very similarly to those with open bottoms, with an average 140 percent increase in estimated  $E$  if the sine function was used to calculate  $(T_o - T_d)$ . By comparison, estimated  $E$  for plastic ML's using calculated  $(T_o - T_d)$  values averaged 104 percent higher than that estimated using actual  $(T_o - T_d)$ . The smaller difference between the two methods of estimation, when data from plastic ML's was used, was at least partially due to the smaller absolute value of  $(T_o - T_d)$  during nighttime for plastic ML's. Estimated  $E$  could not be compared to actual  $E$  since the ML's with thermistors could not be weighed.

Juxtaposition of wind speed (Figure 3-4) with estimated  $E$  (Figures 6-3 and 6-4) for the same 24 hour periods showed the expected positive correlation between daytime evaporation and wind speed. During nighttime, what little correlation existed was negative when actual  $(T_o - T_d)$  values were used for  $E$  estimation.

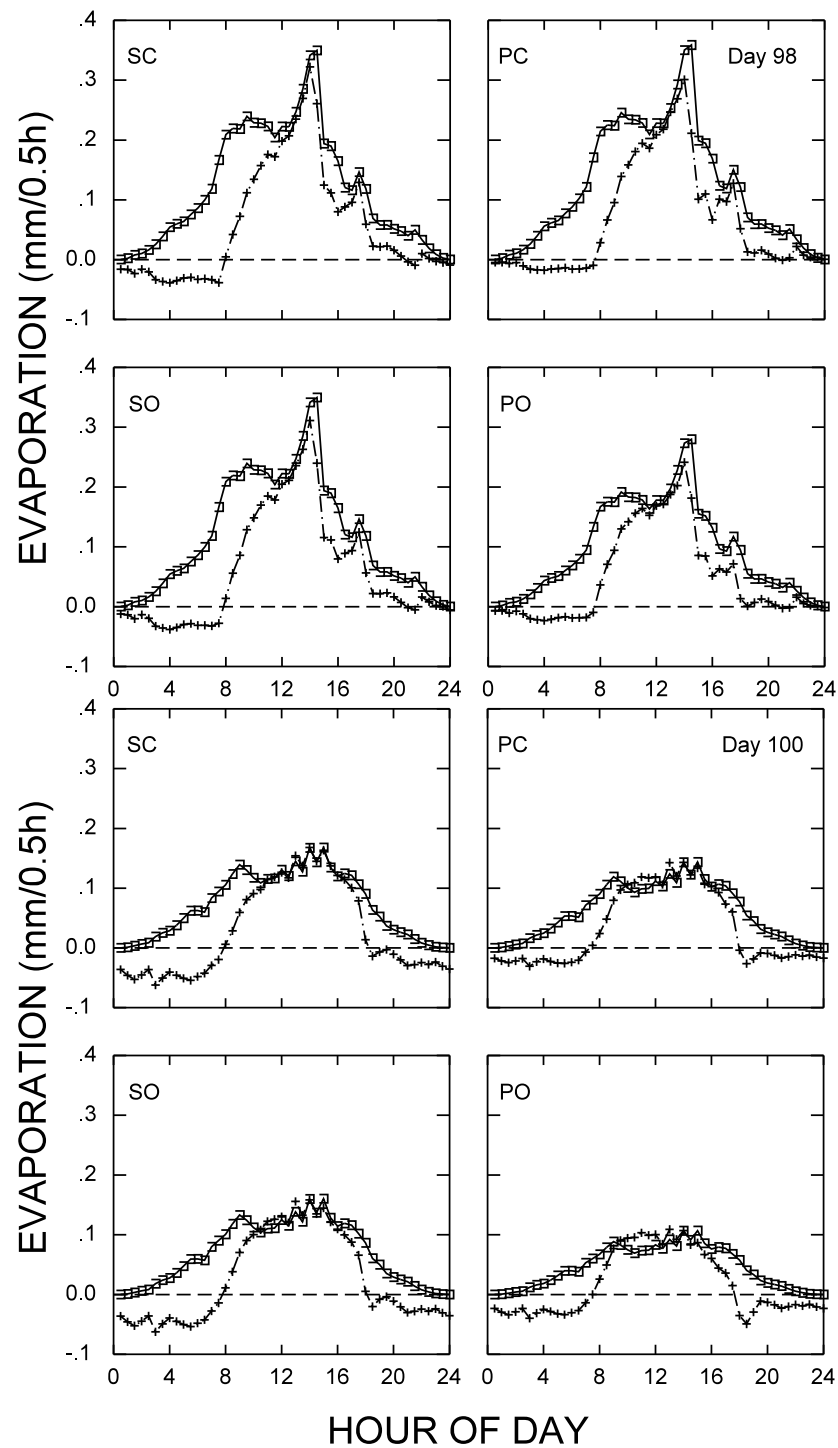
The negative nighttime estimates of  $E$  were unrealistic since calculations showed that the dew-point temperature was

never reached at the surfaces of either the ML's, the dry soil or the two field soil measurement locations during the 9 days of the experiment. The negative E estimates must therefore be due to either differences in soil heat flux or to differences in short wave radiation flux between the dry and drying soils. At night the short wave radiation flux is zero so the negative E estimate is clearly due to differences in the soil heat flux between the dry and drying soils - differences that were assumed to be negligible in comparison to latent heat flux when integrated over 24 hours (Ben-Asher et. al. 1983).

The degree of sensible heat flux can be appreciated by close examination of Figures 3-5 through 3-8. For the nights of days 95-96, 96-97 and 99-100, the difference between air and drying soil temperatures was nil yet the difference between dry and drying soil temperatures was considerable especially for the nights of days 95-96 and 96-97 when the largest negative temperature depressions were recorded (about  $-4^{\circ}\text{C}$  for steel ML's). On other nights, notably those when cloud cover was present, air temperature was several degrees higher than soil temperatures and dry and drying soil temperatures were much closer together. Still, nighttime drying soil temperatures were considerable higher than dry



**Figure 6-3.** Comparison of evaporation estimated using Equation 6-5 (sine wave approximation of  $T_o - T_d$ ) [squares] with that estimated using actual  $T_o - T_d$  [crosses]. Days 93 and 95.



**Figure 6-4.** Comparison of evaporation estimated using Equation 6-5 (sine wave approximation of  $T_o - T_d$ ) [squares] with that estimated using actual  $T_o - T_d$  [crosses]. Days 98 and 100.

soil temperatures and temperatures of steel ML's were consistently higher than those of plastic ML's pointing up the differences in soil heat flux and nighttime sensible heat flux among the ML treatments and the reference dry soil.

One possible solution would be to integrate Equation 6-3 with a half hour time step as before but describing the reference dry soil temperature as (from Equation 5-15):

$$T_o = \bar{T}_o + 0.5(T_{o,max} - T_{o,min})\sin(\omega t) \quad [6-6]$$

where  $\bar{T}_o = 0.5(T_{o,max} + T_{o,min})$  is the diurnal average reference dry soil temperature, and  $T_{o,max}$  and  $T_{o,min}$  are the maximum and minimum reference temperatures, respectively (measured by infrared thermometer). The drying soil temperature would be described separately as:

$$T_d = \bar{T}_d + 0.5(T_{d,max} - T_{d,min})\sin(\omega t) \quad [6-7]$$

where  $\bar{T}_d = 0.5(T_{d,max} + T_{d,min})$  is the diurnal average drying soil temperature, and  $T_{d,max}$  and  $T_{d,min}$  are the drying soil maximum and minimum temperatures, respectively. Using Equations 6-6 and 6-7 would eliminate the assumption that the minimum temperatures of dry and drying soils were equal, and would allow negative temperature depressions.

Integration of Equation 6-3 with  $T_o$  and  $T_d$  described by Equations 6-6 and 6-7, with a half hour time step starting at

7 AM and proceeding for 24 hours, with correction for the time of 1st weighing, and with  $T_m^3$  represented by Equation 6-4, resulted in an  $r^2$  value of 0.551 for regression of  $E_a$  vs.  $E_{est}$  with dummy variables for the treatments. This was essentially equal to the  $r^2$  of 0.549 resulting from a similar regression in the previous section.

Several results are clear. First, the use of actual  $(T_o - T_d)$  values, in place of the temperature depression as estimated using Equation 6-5, should greatly reduce over-estimation by the EBM. Allowing the minimum temperatures of dry and drying soils to take on different values, while continuing to use the sine wave formulation for surface temperature, did not appreciably improve the model. The difference, between dry and drying soils, in nighttime soil heat flux was important and caused the model to predict a false negative evaporation at night. In the model, the effect of outward nighttime soil heat flux may be balanced by that of inward daytime soil heat flux, resulting in a net zero estimation of evaporation due to soil heat flux over a 24 hour period, but only if the model is integrated over a 24 hour period, not over the 12 hour period suggested by Ben-Asher et al. (1983). In other words, the false negative evaporation at night may be balanced by false positive evaporation during the day. Finally, the atmosphere was stable or neutral at night and unstable during the day which suggests that separate

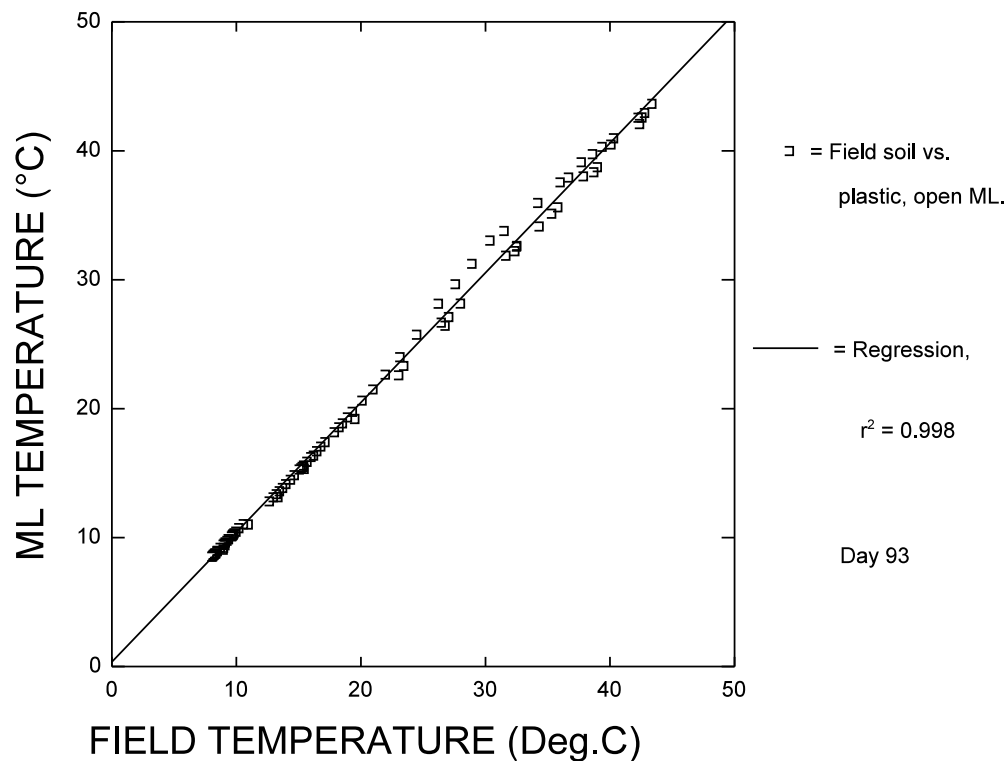
aerodynamic resistance equations should be used, one for stable and one for unstable conditions.

Estimating Temperature Depression.

In the previous section it was shown that the sine wave approximation of soil surface temperature, represented by Equation 5-21, was inaccurate and that actual drying soil and reference dry soil temperatures ( $T_d$  and  $T_o$ , respectively) should be used in the EBM. Since collection of actual ( $T_o - T_d$ ) values on a useful interval (say 1 hour) in the field is either very labor or equipment intensive it is desirable to have some method of estimating ( $T_o - T_d$ ) from intensive automated measurements at one or two locations coupled with extensive measurements at all field locations only once or twice a day. Of course Equation 5-21 represents one such method but has been shown to be insufficient. What is required is a method that would closely predict the actual ( $T_o - T_d$ ) values in all their variation (see Figures 6-1 and 6-2).

From Figures 3-5 through 3-8 it is apparent that the soil temperature measured at a nearby field site closely matched the variability in surface temperatures of the ML's, differing mainly in maximum, minimum and a slight phase shift. Regression of ML temperatures vs. field soil temperatures showed very good correlation for all cases ( $R^2 > 0.99$ , example in Figure 6-5) but as expected the slopes and intercepts were not one and zero, respectively. Were a method available to translate intensive measurements of field soil surface temperature into accurate predictions of ML temperatures on an

hourly or smaller interval, then the problem of accurately estimating ( $T_o - T_d$ ) would devolve to that of estimating dry soil temperature on the same interval.



**Figure 6-5.** Example of linear relationship between surface temperatures of nearby field soil and microlysimeter soil. Measurements at 15 minute intervals.

A linear relationship between field soil temperature and ML temperature was defined in order to convert field soil temperatures (FT) to estimates of ML temperatures,  $T_d$ . Requirements for the relationship were that maximum and minimum estimated ML temperatures should equal the maximum and minimum ML temperatures as measured by IR thermometer ( $MLIR_{max}$ ).

and  $MLIR_{min}$ , respectively). The relationship was:

$$T_d = b_0 + b_1(FT) \quad [6-8a]$$

where

$$b_1 = (MLIR_{max} - MLIR_{min}) / (FL_{max} - FL_{min}) \quad [6-8b]$$

$$b_0 = MLIR_{max} - b_1(FL_{max}) \quad [6-8c]$$

and where  $FL_{max}$  was the maximum field soil temperature measured by thermistor. Also,  $FL_{min}$  was the minimum field soil temperature measured by thermistor.

Equation 6-8 was used to estimate ML temperatures for days 93 and 95 for the steel and plastic ML's which had been instrumented with thermistors. Regression of estimated vs. actual temperature showed very good correlation ( $R^2 > 0.99$ ) for the 8 cases but the slopes and intercepts of the regression lines were not exactly one and zero respectively. This result was expected since in general the maximum and minimum temperatures as measured by IR thermometry were up to 5 degrees higher than those measured by thermistor.

The shape of the temperature curve was very well reproduced (Figure 6-6) and for this reason, and since the estimated maximum and minimum temperatures were equal to the extremes as measured by IRT, the procedure was assumed to accurately predict ML surface temperatures as they would be measured by IR thermometry. The slight phase difference

between estimated and actual temperatures was disregarded.

An exactly analogous procedure was used to estimate reference dry soil temperatures,  $T_o$ , from temperatures, RDST, measured by thermistor in the reference, and from the maximum and minimum reference dry soil temperatures as measured by IR thermometer ( $RDSIR_{\max}$  and  $RDSIR_{\min}$ , respectively):

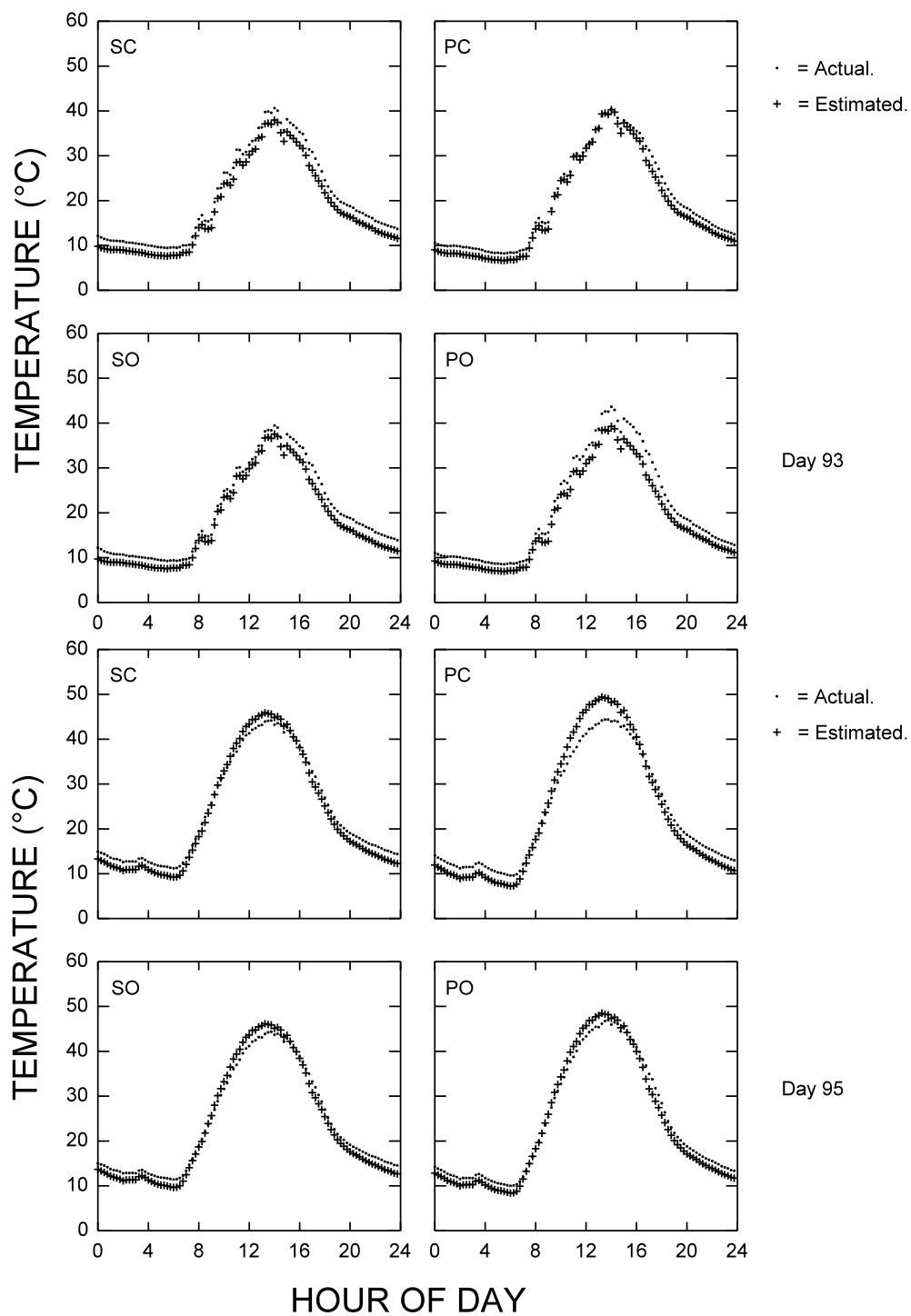
$$T_o = b_0 + b_1 (RDST) \quad [6-9a]$$

where

$$b_1 = (RDSIR_{\max} - RDSIR_{\min}) / (RDST_{\max} - RDST_{\min}) \quad [6-9b]$$

$$b_0 = RDSIR_{\max} - b_1 (RDST_{\max}) \quad [6-9c]$$

and where  $RDST_{\max}$  was the maximum reference dry soil temperature measured by thermistor. Also,  $RDST_{\min}$  was the minimum reference dry soil temperature measured by thermistor.



**Figure 6-6.** Actual ML surface temperatures measured by thermistor vs. those estimated using Equation 6-8. The actual daily temperature curves were well represented.

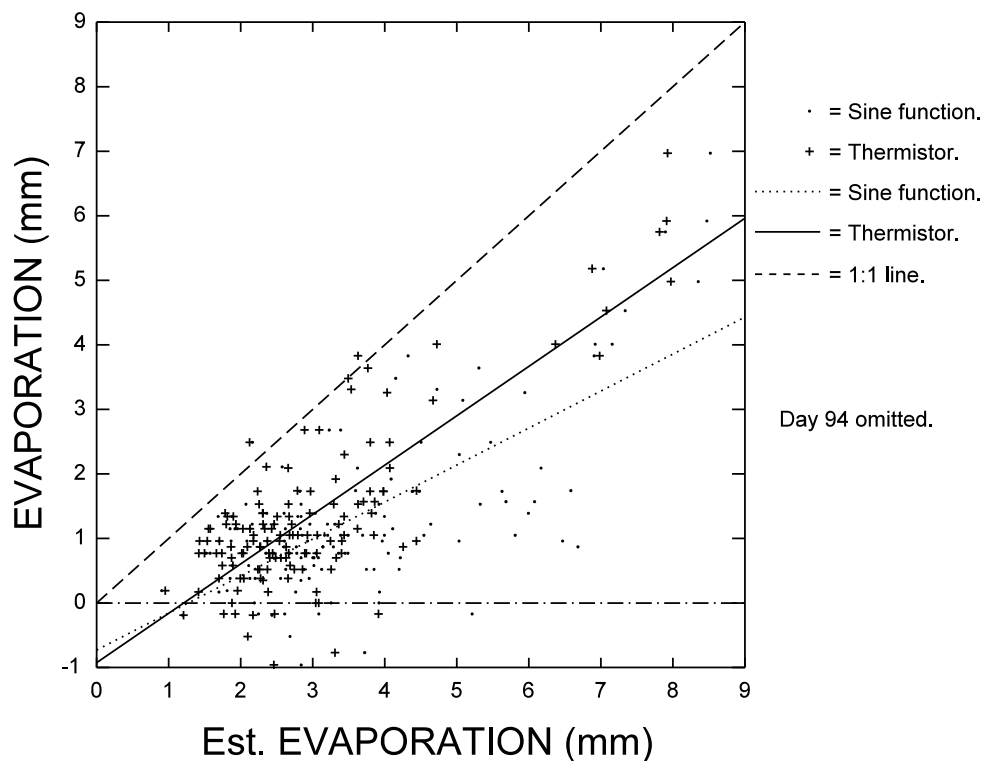
First Improved Energy Balance Model.

A reliable method for estimating soil surface temperatures on a half-hourly basis was demonstrated in the previous section. This method is now used to estimate the temperature depression ( $T_o - T_d$ ) in the EBM, replacing the sine wave approximation represented by Equation 5-21. Once the value of ( $T_o - T_d$ ) was available, it was no longer necessary to use assumptions number 6 [that the quantity ( $T_o^4 - T_d^4$ ) was approximated by  $\bar{T}_m^3(T_o - T_d)$ ], and number 7 [that  $\bar{T}_m^3$  was constant over a 24 h period]. Substituting Equations 5-12 and 5-14 into Equation 5-8 and rearranging resulted in:

$$\int E \, dt = \int [\rho C_p (T_o - T_d) / r + \epsilon \sigma (T_o^4 - T_d^4)] / L_e \, dt \quad [6-10]$$

Using the estimated ML and ref. dry soil surface temperatures, the value of ( $T_o - T_d$ ) was calculated and Equation 6-10 was numerically integrated with a 30 minute time step. Integration started at the time of weighing on the first day after irrigation and was started and stopped at 7 AM on every day thereafter resulting in estimates of evaporation for the periods between weighings of the ML's. Integration was carried out for nighttime as well as daytime but, since the dew point was never reached, only positive values of evaporation were summed.

Regression of actual vs. estimated evaporation with dummy variables for the treatments showed a marked improvement in estimation ( $R^2 = 0.66$  vs.  $R^2 = 0.55$  for  $T_o$  and  $T_d$  based on the sine functions of Equations 6-6 and 6-7) (Figure 6-7, Table 6-3). This model still overestimated evaporation but less so than the model based on Equation 5-23 (Table 6-1). The new energy balance model (called EBM1) consisted of Equations 6-8, 6-9 and 6-10.



**Figure 6-7.** Comparison of evaporation estimated using  $(T_o - T_d)$  calculated with Equation 6-5 (Sine function) ( $r^2 = 0.49$ , simple regression) to that estimated using  $(T_o - T_d)$  based on scaling from field soil temperatures using IR thermometer maxima and minima and Equations 6-8 and 6-9 (Thermistor) ( $r^2 = 0.64$ , simple regression).

**Table 6-3.**

Regression analyses for daily evaporation,  $E_a$ , (mm) with the estimated evaporation,  $E_{est}$ , (mm) from numerical integration of Equation 6-10, as the independent variable; and dummy variables for length and wall type treatments. Aerodynamic resistance,  $r = 126(U)^{-0.96}$ . Summation started at time of first weighing and from 7 AM to 7 AM thereafter. Negative values neglected.

Model:  $E_a = b_0 + b_1 E_{est}$

$r^2 = 0.635$ ,  $n = 136$ .

parameter	estimate	std. error	significance
intercept	-0.924	0.163	0.000
$E_{est}$	0.765	0.050	0.000

Model:  $E_a = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 + b_5 x_5 + b_6 E_{est}$   
 $+ b_{16} x_{16} + b_{26} x_{26} + b_{36} x_{36} + b_{46} x_{46} + b_{56} x_{56}$

See Appendix E for explanation of model.

$r^2 = 0.659$ ,  $n = 136$

parameter	estimate	std. error	significance
intercept	-0.752	0.461	0.105
$x_1$	0.008	0.661	0.990
$x_2$	0.241	0.594	0.685
$x_3$	-0.638	0.602	0.291
$x_4$	0.016	0.569	0.978
$x_5$	-1.056	0.678	0.122
$E_{est}$	0.697	0.132	0.000
$x_{16}$	-0.015	0.201	0.940
$x_{26}$	-0.019	0.191	0.921
$x_{36}$	0.163	0.174	0.349
$x_{46}$	0.015	0.168	0.930
$x_{56}$	0.342	0.191	0.076

Equations:

$E_a = -0.744 + 0.682 E_{est}$ , 10 cm, steel  
 $E_a = -0.511 + 0.678 E_{est}$ , 10 cm, plastic  
 $E_a = -1.390 + 0.860 E_{est}$ , 20 cm, steel  
 $E_a = -0.737 + 0.712 E_{est}$ , 20 cm, plastic  
 $E_a = -1.809 + 1.039 E_{est}$ , 30 cm, steel  
 $E_a = -0.752 + 0.697 E_{est}$ , 30 cm, plastic

### Aerodynamic Resistance/Transfer Coefficients.

The improved energy balance model (EBM1) presented in the previous section still over-estimated evaporation though less so than the original EBM. Also, EBM1 was still rather inaccurate. Examination of the soil to air temperature differences in Figures 3-5 through 3-8, shows that atmospheric conditions were stable at night and unstable during the day. The single aerodynamic resistance equation (Equation 5-11) would not be expected to perform well under both stability conditions, causing inaccuracy. Also, as previously stated, Equation 5-11 may underestimate  $r$  with a resulting over-estimation of  $E$  by the EBM1. In this section the effects of the aerodynamic resistance,  $r$ , on EBM1 will be investigated. Much of the literature deals with the inverse of  $r$ , which is the transfer coefficient,  $D$ . In what follows keep in mind that:

$$D = 1/r \quad [6-11]$$

### Stability Correction.

The transfer coefficient can be corrected with equations that include the soil to air temperature difference ( $T_s - T_a$ ). Kreith and Sellers (1975) published equations for the transfer coefficient over bare soil in Arizona. For neutrally stable conditions the transfer coefficient for sensible heat flux,  $D_h$ , is equal to that for momentum flux,  $D_m$ :

$$D_h = D_m = k^2 u (\ln(z/z_o))^{-2} = 0.0022 u \quad [6-12]$$

where  $k$  is von Karmann's constant = 0.4,  $u$  is the horizontal wind speed [m/s] at the height  $z$ , and  $z_o$  is the roughness length in the same units as  $z$ . The roughness length was taken to be 0.03 cm for bare soil, as they did.

For unstable conditions the value of  $D_h$  may be several times that of  $D_m$  and was given by Kreith and Sellers as:

$$D_h = D_m (1 + 14 (T_s - T_a) / u^2)^{1/3} \quad [6-13]$$

where  $u$  is in m/s and is measured at 2 m and  $(T_s - T_a)$  is in °C. For stable conditions  $D_h$  is smaller than  $D_m$  and was given by:

$$D_h = D_m (1 - 14 (T_s - T_a) / u^2)^{-1/3} \quad [6-14]$$

The transfer coefficient can also be given in terms of the resistance used by Ben-Asher et al. (1983) by inverting Equation 5-11:

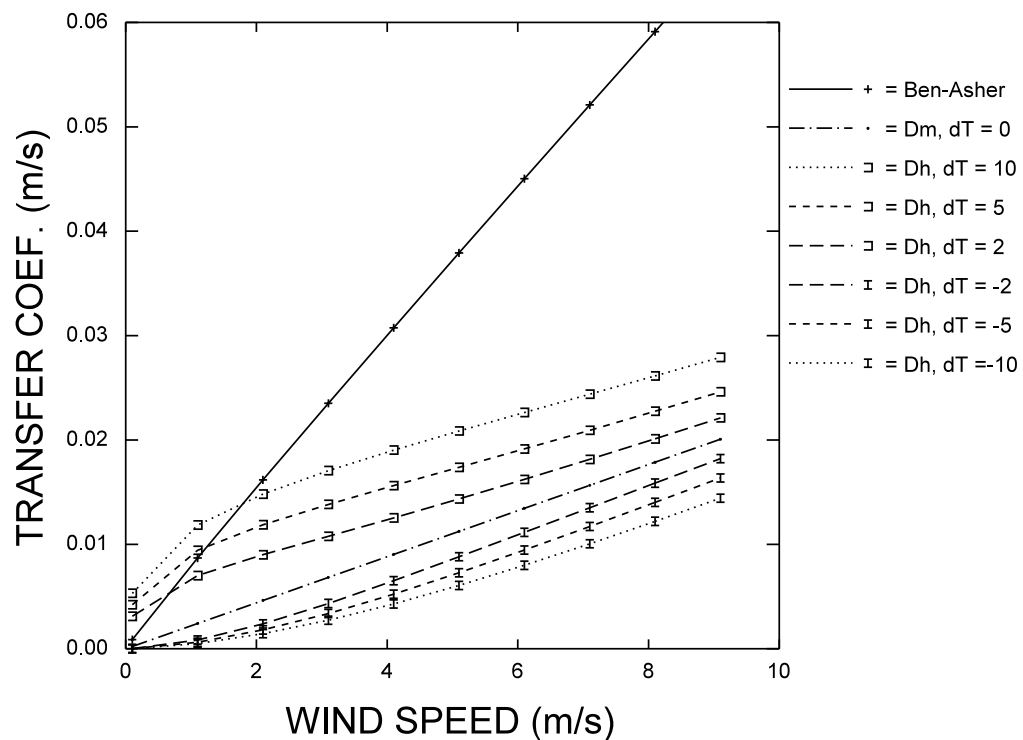
$$D_h = 1/r = 0.0079 u^{0.96} \quad [6-15]$$

Values of  $D_h$  calculated from equations 6-12, 6-13, 6-14 and 6-15 were plotted versus wind speed for several different values of  $(T_s - T_a)$  (Figure 6-8). Equation 5-11 overpredicts the transfer coefficient for bare soil as would be expected since it originated from data for sugar beets. For example,

the roughness length for crops is empirically given as (Rosenberg et al. 1983, Eq. 4.4):

$$\log_{10} z_o = 0.997 \log_{10} h - 0.883 \quad [6-16]$$

where  $z_o$  and  $h$  are in meters and  $h$  is the crop height. For a 30 cm tall sugar beet crop Equation 6-16 gives  $z_o = 3.9$  cm and, for wind speed measured at 2 m, Equation 6-12 reduces to:



**Figure 6-8.** Comparison of the transfer coefficient (inverse of the aerodynamic resistance) used by Ben-Asher et al. (1983) with that given by Kreith and Sellers (1975) for bare soil.  $D_m$  is for neutral atmospheric conditions;  $D_h$  is for unstable conditions [squares] or stable conditions [I's];  $dT$  is the soil - air temperature difference.

$$D_m = 0.010 u \quad [6-17]$$

which is reasonably close to the inverse of Equation 5-11.

The inverse of Equation 5-11 over-predicted the transfer coefficient for sensible heat flux when compared to the stability corrected transfer coefficients for bare soil published by Kreith and Sellers (1975) except at wind speeds less than 1 m/s and  $(T_s - T_a)$  greater than zero. Here, it will be shown that the stability corrected transfer coefficients (Equations 6-12, 6-13 and 6-14) were not a good replacement for the factor  $1/r$  in EBM1 (Equation 6-10). However the transfer coefficient for neutral stability, Equation 6-12, does prove to be a good replacement for  $1/r$  and leads to an improved EBM.

In order to use stability correction the EBM must be re-written with separate terms for sensible heat flux from dry and drying soils:

$$\begin{aligned} \int E \, dt = \int [ & \rho C_p (T_o - T_a) D_{h,o} - \rho C_p (T_d - T_a) D_{h,d} \\ & + \epsilon \sigma (T_o^4 - T_d^4) ] / L_e \, dt \end{aligned} \quad [6-18]$$

where  $T_a$  was the air temperature at 1.5 m. Equations 6-12, 6-13 and 6-14 were used to define  $D_{h,o}$  with  $T_o$  replacing  $T_s$  and the same equations were used to define  $D_{h,d}$  with  $T_d$  replacing  $T_s$ . The resulting model consisted of Equations 6-8, 6-9, 6-12, 6-13 and 6-14, and 6-18. This model was numerically

integrated with a 30 minute time step. Integration was started at the time of weighing on the first day after irrigation and was started and stopped at 7 AM on every day thereafter. Integration was carried out for nighttime as well as daytime but negative values of evaporation were not summed.

Regressing actual evaporation,  $E_a$ , against the predicted evaporation,  $E_{est}$ , resulted in the regression equation:

$$E_a = -1.26 + 0.87 E_{est} \quad [6-19]$$

The  $r^2$  value was lower (0.57 vs. 0.64, for simple regression) than obtained in the previous section and the intercept was farther from zero. The more negative intercept was caused by increased overestimation of evaporation for later days when evaporation was small. Conversely, on the first day after irrigation, estimates of evaporation were less than those found in the previous section.

Consideration of the field placement of sensors, and of the reference dry soil, provides insight into the poor performance. Equations 6-12, 6-13, and 6-14 are valid only within the internal boundary layer, a layer extending from the ground upward within which the momentum flux should be independent of height and within which a logarithmic wind profile, characteristic of the underlying surface, should develop (Rosenberg et al. 1983). This is because the derivation of Equation 6-12 assumes the existence of a

logarithmic wind profile which does not change from place to place in the field. The temperature difference ( $T_s - T_a$ ) is only physically meaningful under these conditions since computation of sensible heat flux by Equations 5-9 and 5-10 implies an aerodynamic resistance or transfer coefficient that is single valued from the soil to the point of air temperature measurement.

Air temperature was measured at a weather station at the east end of the field and was considered to be a good representation of the air temperature within the internal boundary layer over the drying soil. Because the reference dry soil was in a small container (29 cm diameter), isolated in the field of drying soil, the measured air temperatures should not be used in Equations 6-13 and 6-14 to calculate  $D_h$  for the reference dry soil. In fact the thickness of the fully adjusted layer over the reference dry soil was only about 0.5 cm (Rosenberg et al. 1983, Equation 4.7) so the air temperature relative to the reference would have to be measured at less than 5 mm height in order to be useful in Equations 6-13 and 6-14.

In day time the reference dry soil was a small, relatively hot, disk surrounded by a surface that was always cooler and with air flow above which was always cooler. These circumstances suggest that buoyancy effects may have dominated in sensible heat transfer from the reference soil, in which

case the effect of wind speed on the transfer coefficient might be reduced.

Transfer Coefficient for Neutral Conditions and  
Second Improved Energy Balance Model.

The stability corrected transfer coefficients proved to work poorly. One method of addressing this problem would be to use only Equation 6-12 to calculate the transfer coefficient,  $D_h$ , ignoring atmospheric stability conditions. Combining sensible heat flux terms in Equation 6-18 gave:

$$\int E \, dt = \int [\rho C_p (T_o - T_d) D_h + \epsilon \sigma (T_o^4 - T_d^4)] / L_e \, dt \quad [6-20]$$

This model (call it EBM2), consisting of Equations 6-8, 6-9, 6-20 and 6-12, was integrated under the conditions stated in the previous section. Regressing actual evaporation,  $E_a$ , against the predicted evaporation,  $E_{est}$ , resulted in the regression equation:

$$E_a = -0.84 + 1.60 E_{est} \quad [6-21]$$

with  $r^2 = 0.67$ . Regression using dummy variables for the ML treatments resulted in an  $R^2$  of 0.69 (Table 6-4). This model was more accurate than EBM1 or the original EBM. Note that the regression lines for steel and plastic ML's were a great deal different (Table 6-4). When a dummy variable was used only to differentiate wall material the regression line for

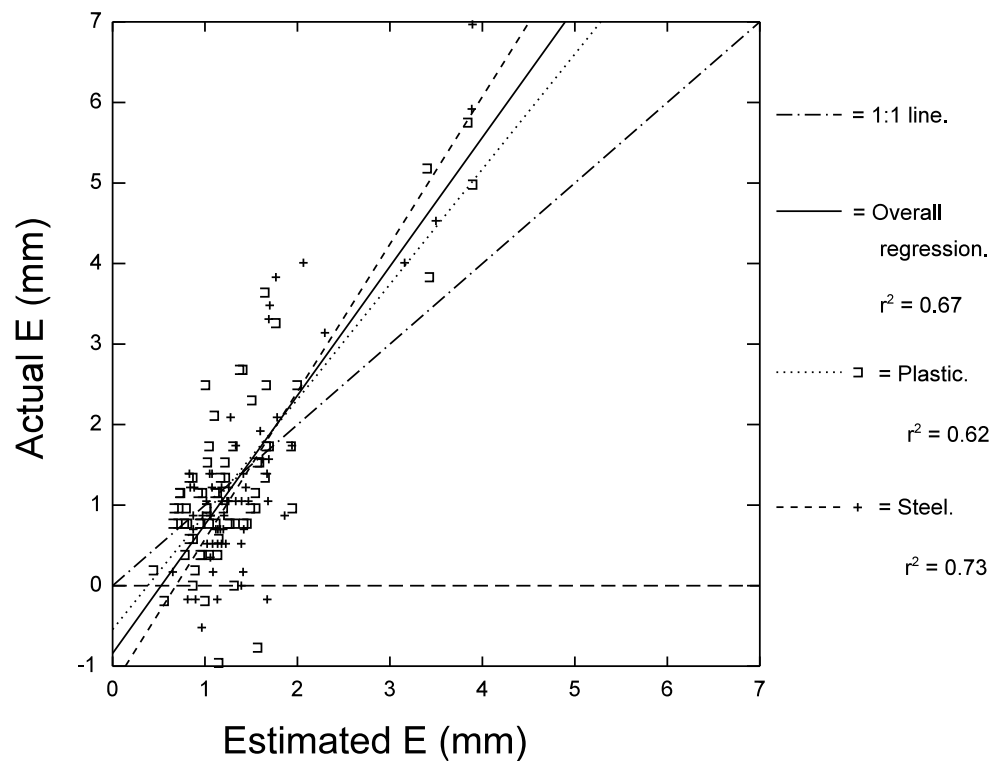
plastic ML's was (Figure 6-9):

$$E_a = -0.55 + 1.43 E_{est} \quad [6-22]$$

The corresponding equation for steel ML's was:

$$E_a = -1.25 + 1.83 E_{est} \quad [6-23]$$

The slopes of these lines were significantly different at the 5 percent level of probability.



**Figure 6-9.** Regression of actual evaporation vs. that estimated using Equation 6-20 numerically integrated from 7 AM to 7 AM, using the transfer coefficient for neutral atmospheric conditions (Equation 6-12).

Obviously the model did not account for all physical differences between ML's. The difference is probably accounted for by a combination of the greater heat flux in steel ML's and the differences in sensible heat flux between plastic and steel ML's (sensible heat flux from exposed edges of steel ML's might be significant but would not be reflected by the model). At 1.60, the slope for the regression of  $E_a$  vs.  $E_{est}$  was the largest of any found so far. The model was no longer overestimating but now underestimated evaporation, at least when evaporation rates were high. The  $R^2$  value of 0.69 (for regression of  $E_a$  vs.  $E_{est}$  with dummy variables) represented a considerable improvement over the  $R^2$  of 0.55 found for the original model.

**Table 6-4.**

Regression analyses for daily evaporation,  $E_a$ , (mm) with the estimated evaporation,  $E_{est}$ , (mm) from numerical integration of Equation 6-20, as the independent variable; and dummy variables for length and wall type treatments. Transfer coefficient given by Equation 6-12. Summation from 7 AM to 7 AM but started at time of first weighing. Negative values neglected.

Model:  $E_a = b_0 + b_1 E_{est}$

$r^2 = 0.668$ ,  $n = 136$ .

parameter	estimate	std. error	significance
intercept	-0.842	0.148	0.000
$E_{est}$	1.602	0.098	0.000

Model:  $E_a = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 + b_5 x_5 + b_6 E_{est}$   
 $+ b_{16} x_{16} + b_{26} x_{26} + b_{36} x_{36} + b_{46} x_{46} + b_{56} x_{56}$

See Appendix E for explanation of model.

$r^2 = 0.694$ ,  $n = 136$

parameter	estimate	std. error	significance
intercept	-0.672	0.415	0.108
$x_1$	-0.100	0.596	0.868
$x_2$	0.215	0.536	0.690
$x_3$	-0.554	0.540	0.308
$x_4$	0.071	0.512	0.890
$x_5$	-1.102	0.612	0.074
$E_{est}$	1.463	0.257	0.000
$x_{16}$	0.038	0.390	0.923
$x_{26}$	-0.032	0.371	0.932
$x_{36}$	0.286	0.334	0.394
$x_{46}$	-0.013	0.325	0.967
$x_{56}$	0.767	0.371	0.041

Equations:

$E_a = -0.772 + 1.501 E_{est}$ , 10 cm, steel  
 $E_a = -0.458 + 1.431 E_{est}$ , 10 cm, plastic  
 $E_a = -1.226 + 1.749 E_{est}$ , 20 cm, steel  
 $E_a = -0.601 + 1.450 E_{est}$ , 20 cm, plastic  
 $E_a = -1.774 + 2.231 E_{est}$ , 30 cm, steel  
 $E_a = -0.672 + 1.463 E_{est}$ , 30 cm, plastic

Empirical Transfer Coefficient and  
Third Improved Energy Balance Model.

Use of Equation 6-12 to calculate the transfer coefficient for sensible heat flux produced the best results so far from the EBM. However, Equation 6-12 assumed the presence of a logarithmic wind profile between the points of measurement of air temperature and soil temperature, a condition which did not exist in the case of the reference dry soil. It was possible that an empirical fit of the transfer coefficient function would improve model performance by better reflecting the conditions of the study.

An empirical transfer coefficient function for the reference dry soil,  $D_{h,o}$ , was defined, as in Equation 6-15, in terms of the wind speed,  $u$ , [m/s] measured at 1.5 m:

$$D_{h,o} = c_0 u^{c_1} \quad [6-24]$$

where  $c_0$  and  $c_1$  were fitted coefficients. The transfer coefficient for drying soil,  $D_{h,d}$ , was taken equal to that for neutral conditions (Equation 6-12). The coefficients used in the original EBM were  $c_0 = 0.00794$  and  $c_1 = 0.96$  for  $D_m$  in m/s. The EBM thus consisted of Equations 6-8, 6-9, 6-18 and 6-12 with Equation 6-24 describing the transfer coefficient for dry soil.

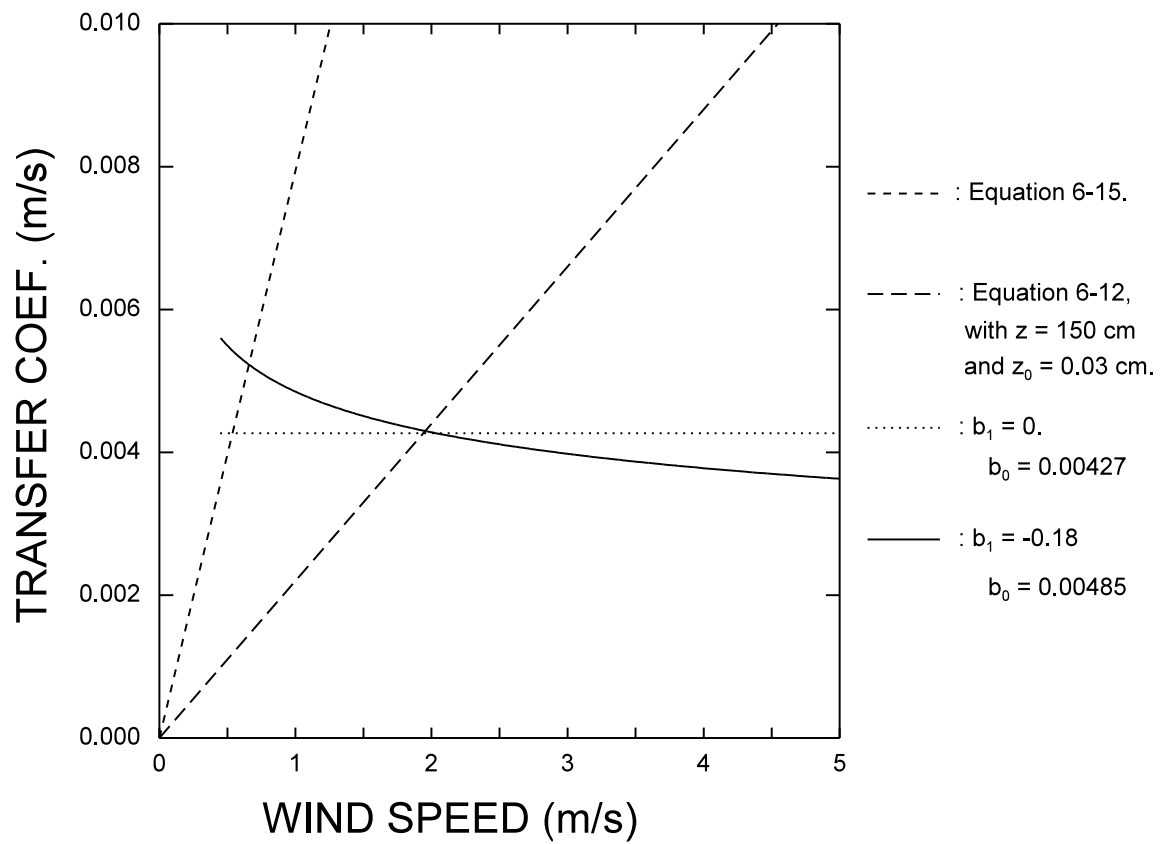
A simple search was done for the "best" coefficients by varying both coefficients over a range of values, using the

EBM to estimate E for all ML's on all days (9 days, 17 ML's) for each pair of coefficient values and calculating the  $r^2$  and sum of squared error, SSE, from regression of actual versus estimated E, for each pair of coefficient values. The range of values was 0.0500 to 0.0013 for  $c_0$  and 1.00 to -0.2 for  $c_1$ . The pair of coefficients resulting in the lowest SSE were taken as the "best" coefficients. The model was integrated with a 30 minute time step. Integration began at the time of first weighing and was started and stopped at 7 AM on every day thereafter. Negative values of evaporation were not summed.

The lowest value of SSE was 68.04 for  $c_0 = 0.00485$  and  $c_1 = -0.18$ . A negative exponent in Equation 6-24 would give a value of  $D_{h,o} = \infty$  for zero wind speed, clearly an unrealistic result but one made possible by the fact that the anemometer calibration used always gave wind speeds at or above 0.447 m/s. When  $c_1$  was limited to zero or positive values in the search the lowest value of SSE was 68.12 for  $c_0 = 0.00427$  and  $c_1 = \text{zero}$ . Both sets of values for  $c_0$  and  $c_1$  gave practically the same values of  $D_{h,o}$  when used in Equation 6-24 (Figure 6-10), and gave nearly identical results when used in the EBM. The latter set with zero exponent was chosen for use in the EBM since it is more realistic at low wind speeds. Thus a new model (call it EBM3) consisted of Equations 6-8, 6-9 and 6-18 with  $D_{h,d}$  described by Equation 6-12 and  $D_{h,o}$  described by:

$$D_{h,o} = 0.00427 \quad u^0 = 0.00427$$

[6-25]



**Figure 6-10.** Transfer coefficient for sensible heat flux as predicted by two functions from the literature and the best fit functions from Experiment 2 data.

Using EBM3 to estimate evaporation and regressing  $E_a$  versus  $E_{est}$  resulted in:

$$E_a = -0.378 + 1.419 E_{est} \quad [6-26]$$

with  $r^2 = 0.704$ . For regression of  $E_a$  versus  $E_{est}$ , with dummy variables for the treatments, the  $R^2$  was 0.734, the highest so far attained (Table 6-5, Figure 6-11).

Regression of  $E_a$  vs.  $E_{est}$  for steel wall type only resulted in:

$$E_a = -0.690 + 1.62 E_{est} \quad [6-27]$$

with  $r^2 = 0.79$ , and a similar regression for plastic gave:

$$E_a = -0.135 + 1.25 E_{est} \quad [6-28]$$

with  $r^2 = 0.63$ . Regression lines found for the 3 lengths showed that the slopes for plastic ML's were similar in value and the intercepts generally closer to zero than for steel (Table 6-5). This model appears to account for the important differences between plastic ML's of different lengths. Although the range of slopes and intercepts for the 3 lengths of steel ML's is still large, these differences may be due to heat flux for which the model would not be expected to account. The model underestimated evaporation on 4 of 8 days, tending to underestimate evaporation early on and overestimate on the later days after irrigation (Table 6-6).

**Table 6-5.**

Regression analyses for daily evaporation,  $E_a$ , (mm) with the estimated evaporation,  $E_{est}$ , (mm) from numerical integration of Equation 6-18 as the independent variable; and dummy variables for length and wall type treatments. Transfer coefficient for reference dry soil,  $D_{h,o} = 0.00427$ .

Model:  $E_a = b_0 + b_1 E_{est}$

$r^2 = 0.704$ ,  $n = 136$ .

parameter	estimate	std. error	significance
intercept	-0.378	0.114	0.001
$E_{est}$	1.419	0.079	0.000

Model:  $E_a = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 + b_5 x_5 + b_6 E_{est}$   
 $+ b_{16} x_{16} + b_{26} x_{26} + b_{36} x_{36} + b_{46} x_{46} + b_{56} x_{56}$

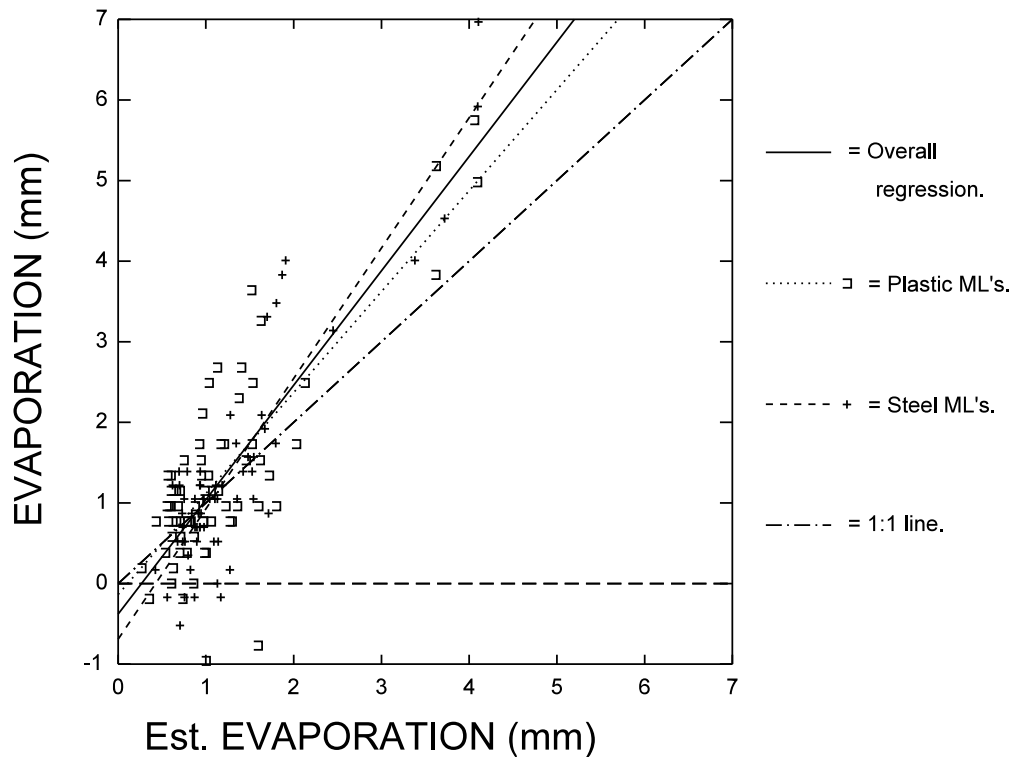
See Appendix E for explanation of model.

$r^2 = 0.734$ ,  $n = 136$

parameter	estimate	std. error	significance
intercept	-0.236	0.325	0.468
$x_1$	-0.077	0.446	0.864
$x_2$	0.199	0.413	0.630
$x_3$	-0.399	0.418	0.340
$x_4$	0.058	0.397	0.885
$x_5$	-0.944	0.471	0.047
$E_{est}$	1.287	0.210	0.000
$x_{16}$	0.027	0.308	0.930
$x_{26}$	-0.077	0.300	0.799
$x_{36}$	0.218	0.270	0.421
$x_{46}$	-0.012	0.264	0.964
$x_{56}$	0.772	0.300	0.011

Equations:

$E_a = -0.313 + 1.315 E_{est}$ , 10 cm, steel  
 $E_a = -0.037 + 1.211 E_{est}$ , 10 cm, plastic  
 $E_a = -0.636 + 1.506 E_{est}$ , 20 cm, steel  
 $E_a = -0.179 + 1.275 E_{est}$ , 20 cm, plastic  
 $E_a = -1.180 + 2.059 E_{est}$ , 30 cm, steel  
 $E_a = -0.236 + 1.287 E_{est}$ , 30 cm, plastic



**Figure 6-11.** Evaporation from ML's regressed against predictions from model using Equation 6-18. Corrected for time of first weighing.  $R^2 = 0.734$  for model with dummy variables for ML treatments. Exchange coefficient for reference dry soil,  $D_{h,o} = 0.00427$ .

The value of zero for the exponent in Equation 6-25 indicates that wind speed had negligible effect on sensible heat flux from the dry soil. This result supports the idea that buoyancy effects were of much greater importance for the reference dry soil than for the field as a whole. Equation 6-25 can be considered the dry soil transfer coefficient function for unstable conditions since only positive half

**Table 6-6.**

Average daily estimated evaporation ( $E_{\text{est}}$ ) from EBM3, average daily evaporation measured by ML's ( $E_{\text{ML}}$ ) and the difference (Diff) (mm).

Day	92	93	95	96	97	98	99	100
$E_{\text{est}}$	2.80	1.56	0.98	0.81	1.24	0.86	0.73	0.68
$E_{\text{ML}}$	3.62	1.93	0.91	1.12	1.20	0.74	0.93	0.23
Diff	-0.82	-0.37	0.07	-0.31	0.04	0.12	-0.20	0.43

hourly values of evaporation were summed while finding the best fit coefficients. For the most part positive values occurred when the air was unstable.

### Reprise of Model Assumptions.

Of the 11 assumptions originally made for the model the following were rejected or replaced. The sine function approximation of soil surface temperatures (assumption 5) was rejected since it resulted in large overestimates of the  $(T_o - T_d)$  term. Estimates of  $T_o$  and  $T_d$  based on thermistor and IR thermometer measurements were generated on a half-hourly basis replacing the sine function. The assumption that  $T_{o,min} = T_{d,min}$  (assumption 7) was shown to be incorrect by up to 4 °C and was no longer needed once the  $T_o$  and  $T_d$  estimates were available. The assumption that  $[(T_o^4 - T_d^4) = \bar{T}_m^3(T_o - T_d)]$  (assumption 6) was shown to be a very good approximation if  $\bar{T}_m^3$  was taken as the instantaneous average of dry and drying soil temperatures. However, the assumption that  $\bar{T}_m^3$  was constant over a 12 h period (assumption 8) was shown to cause an error of about 15%. Both assumptions 6 and 8 were no longer necessary once half-hourly temperature estimates were available. The assumption that wind speed was constant during daytime (assumption 9) was shown to introduce large errors in prediction and was rejected. Instead, half-hourly average wind speeds were used.

The assumption that all energy flux terms were in phase with soil heat flux (assumption 10) is questionable. Latent heat flux, solar radiation and soil surface temperature were obviously in phase and the first two were positive during

daytime (Figures 6-4, 4-5 and 3-5 through 3-8). While in theory soil heat flux may peak as much as 3 hours earlier than soil temperature (Ben-Asher et al. 1983), data presented in Chapter 4 showed that soil heat flux became positive at about the same time as the radiation fluxes but peaked and became negative sooner, i.e. before sunset (Figure 4-5). Also, the assumption that latent heat flux was negligible except for the hours during which soil heat flux was positive (assumption 11) is questionable. However, both assumptions 10 and 11 were unnecessary once integration was performed over a 24 hour rather than a 12 hour period.

Assumption 1, embodied in Equation 5-7:

$$\int [G_o - G_d + K_{in}(\alpha_o - \alpha_d)] dt \ll \int L_e E dt$$

may be more likely to hold if integration is performed over the full diurnal period since the soil heat flux term will balance to zero over 24 hours unless there is gross heating or cooling of the soil. However, in this study gross cooling of the soil by irrigation water did occur. Moreover, all three improved energy balance models (EBM1, EBM2 and EBM3) performed better if negative values of evaporation were neglected during integration. This means that performance was better if integration was performed only during daytime, i.e. over about a 10 hour period.

For the half of the diurnal period during which soil heat flux is positive, Fox (1968) showed the plausibility of Equation 5-7 based on the possibility that the greater positive heat flux in drying soil would be balanced by the larger amount of radiative energy available to the drying soil due to its lower albedo. This is possible if integration is for the period of positive heat flux and for times when the surface of the drying soil is moist. After first stage drying has ended, the soil surfaces have nearly the same albedo. Consequently the radiative terms in 8 would balance although the heat flux term would likely not balance over periods of positive soil heat flux. Thus, for integration over 12 hours, the assumption loses plausibility when first stage drying ends (after the second day in this experiment). For integration over 24 hours the assumption is plausible for all times after first stage drying ends due to the facts that 1) the individual soil heat flux terms may each balance to zero; and, 2) the radiative term should be zero due to similar albedos. Integration of the L.H.S. of Equation 5-7 over 24 hours during first stage drying would likely result in a non-zero sum due to the inequality of albedos, but the assumption might still be plausible if the latent heat flux term were much larger than the combined heat flux and short wave radiation terms during first stage drying. In the next section it will be shown that neither argument holds for the conditions of this

study.

At this point only 4 assumptions (numbers 1 through 4 above) were necessary for the model. No data were collected that would allow challenge to the assumption that aerodynamic resistance was stationary in space (assumption 3) but there may be a problem with using the quantity  $(T_o - T_a)$  for calculating sensible heat flux from the reference dry soil when the air temperature,  $T_a$ , was measured outside of the internal boundary layer existing over the bucket holding the reference. The aerodynamic resistance equation (number 2, Equation 5-11) was replaced by the more appropriate Equations 6-12 and Equation 6-25 which improved the model fit. Assuming that soil emissivity was constant at 0.95 (assumption 4) would introduce little error as emissivity varied within the expected range of 0.98 to 0.95. The assumption, that the transfer coefficients may be given by Equations 6-12 and 6-25, may be somewhat inaccurate. But, the first assumption, represented by Equation 5-7, is now the assumption most likely to lead to model inaccuracy. The following section will examine the soil heat flux and shortwave radiation terms in Equation 5-7.

### Neglected Terms in Energy Balance Model.

At this point three steps have been taken to improve the EBM. First, integration was changed from a daily basis to a half-hourly time step thus improving the interaction between wind speed, which determines the transfer coefficient for sensible heat flux, and soil temperature which is related to the energy available for sensible, latent, longwave and soil heat fluxes. Second, an accurate method for predicting surface temperatures in ML's and the reference dry soil was demonstrated. The method, consisting of Equations 6-8 and 6-9 was used to substitute for Equation 5-21 (which was the sine wave approximation for surface temperatures) resulting in EBM1. Finally, the transfer coefficients for sensible heat flux from dry and drying soils were found to be better represented by Equations 6-25 and 6-12, respectively, resulting in EBM3 which consisted of Equations 6-18, 6-12, 6-25, 6-8 and 6-9.

These improvements lead to an  $R^2$  value of 0.73 when  $E_{est}$  from EBM3 was regressed against  $E_a$ , with dummy variables to eliminate the effects of ML treatments. The same regression using  $E_{est}$  from the original EBM gave an  $R^2$  of only 0.55. EBM3 is still somewhat imprecise (see point scatter in Figure 6-11) and tends to underestimate evaporation immediately after irrigation and overestimate it later on.

The remaining assumptions likely to cause model inaccuracy are that the transfer coefficients can be given by Equations 6-25 and 6-12, and the assumption that the soil heat flux and reflected shortwave radiation terms in Equation 5-7 are negligible. The following subsections will examine first the soil heat flux term, then the solar radiation term, and finally the combined effect of these terms in Equation 5-7.

#### Soil Temperature and Heat Flux.

If there is no net soil warming or cooling then the diurnal net soil heat flux is zero, a priori. In an irrigated field it is unlikely that such a condition would exist since the addition of large amounts of water can greatly change both soil temperature and heat capacity. Considerable differences between plastic and steel ML's in net soil heat flux were shown in Chapter 4 where it was also shown that net heat flux in plastic ML's was close in magnitude to that in the field soil. In fact the largest net heat fluxes calculated for plastic ML's and field soil were identical at 0.84 mm per day. In this section estimates will be made of the heat flux in a dry soil so that an evaluation of the heat flux term in Equation 5-7 may be made.

For a dry soil, the daily net soil heat flux should be that amount associated with the annual cycle of soil heating and cooling plus any daily deviations due to cloudiness and

air temperature changes. Matthias and Warrick (1987) presented equations describing the annual subsurface soil temperature at 5 cm under bare soil for two locations in Arizona. The Yuma location was quite dry, receiving only 65 mm of rain during the year of the study, and the silty clay soil was similar to that in the present study. Annual air temperature and insolation patterns are similar at Yuma and Marana. Ignoring the diurnal cycle, the annual cycle of temperature at Yuma was well represented ( $R^2 = 0.975$ ) by two harmonics:

$$T(x,t) = A_0 + A_1 \exp(-(x-x')d_{y1}) \sin (wt + B_1 - (x-x')d_{y1}) \\ + A_2 \exp(-(x-x')d_{y2}) \sin (2wt + B_2 - (x-x')d_{y2})$$

[6-29]

where,  $A_0$  is the annual mean temperature at depth  $x'$ ,  $A_1$  and  $A_2$  are the amplitudes of temperature for terms 1 and 2,  $B_1$  and  $B_2$  are the respective phase angles at depth  $x'$ , the damping depths in cm are:

$$d_{y1} = (\pi / (365\alpha))^{1/2} \\ d_{y2} = (2\pi / (365\alpha))^{1/2}$$

$\alpha$  is the thermal diffusivity [ $\text{cm}^2 \text{d}^{-1}$ ],  $t$  is time [d], and  $w = 2\pi/365$ . For mean daily temperature at  $x'=5$  cm,  $A_0=26.40$ ,  $A_1=10.33$ ,  $A_2=1.42$ ,  $B_1=4.32$ , and  $B_2=5.95$ . Over a one year period, the effective thermal diffusivity was found to average

110 cm<sup>2</sup> d<sup>-1</sup> with std. dev. = 80 cm<sup>2</sup> d<sup>-1</sup>.

Equation 6-29 is a solution to the diffusion equation for heat conduction in one dimension (Equation 4-1, Chapter 4) The soil heat flux,  $G$ , is given by Equation 4-2, Chapter 4 as  $G = -k \partial T / \partial x$  where  $k$  is the thermal conductivity [J d<sup>-1</sup> cm<sup>-1</sup> K<sup>-1</sup>]. This is Fourier's law of heat conduction for constant conductivity. Differentiating Equation 6-29 with respect to  $x$  ( $x$  is positive downward) gives

$$\begin{aligned} \partial T / \partial x = & -A_1 d_{y1} \exp(-(x-x')d_{y1}) \sin(\omega t + B_1 - (x-x')d_{y1} + \pi/4) \\ & -A_2 d_{y2} \exp(-(x-x')d_{y2}) \sin(2\omega t + B_2 - (x-x')d_{y2} + \pi/4) \end{aligned}$$

[6-30]

Replacing  $\partial T / \partial x$  in Equation 4-2 with the R.H.S. of Equation 6-30, letting  $x = 0$ , and using the amplitudes and phase angles given above, we have the soil heat flux at the surface:

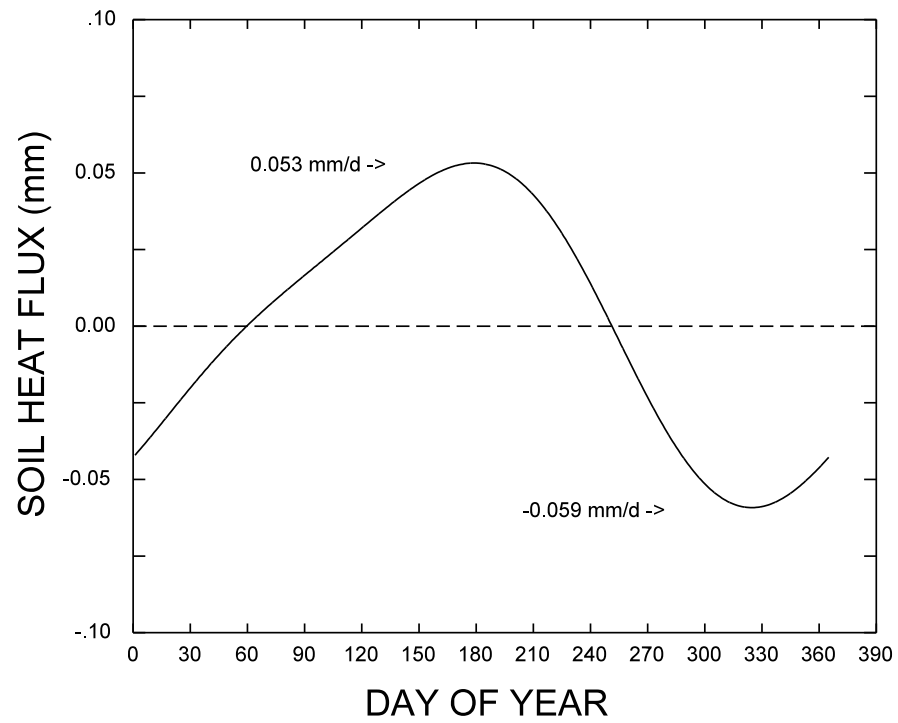
$$\begin{aligned} G = & 10.33 k d_{y1} \exp(5d_{y1}) \sin(2\pi t/365 + 4.32 + \pi/4 + 5d_{y1}) \\ & + 1.42 k d_{y2} \exp(5d_{y2}) \sin(4\pi t/365 + 5.95 + \pi/4 + 5d_{y2}) \end{aligned}$$

[6-31]

Assuming an average water content of 0.05 m<sup>3</sup> m<sup>-3</sup> the heat capacity was evaluated as 1.23 MJ m<sup>-3</sup> K<sup>-1</sup> using Equation 4-4. Combining this with the effective diffusivity of 110 cm<sup>2</sup> d<sup>-1</sup> gives a thermal conductivity of 135 J cm<sup>-1</sup> d<sup>-1</sup> K<sup>-1</sup>. Evaluation of Equation 6-31 using  $k = 135$  J d<sup>-1</sup> cm<sup>-1</sup> K<sup>-1</sup> and  $\alpha = 110$  cm<sup>2</sup> d<sup>-1</sup> shows that the daily net soil heat flux, resulting from the

annual temperature cycle, never exceeds an absolute value equivalent to 0.06 mm of water (Figure 6-12). In early April the calculated daily net heat flux was about 0.02 mm per day, a negligible amount.

Diurnal deviations from the annual temperature cycle in dry soil may be caused by phenomena including cloudiness and regional air temperature changes. In Chapter 4 the daily net soil heat flux in drying soil was calculated to average 0.67 mm with a low of 0.26 mm on the day after irrigation and a high of 0.84 mm. This leads to the conclusion that the heat flux term in Equation 5-7 may have been as large as 0.8 mm per day.



**Figure 6-12.** Daily net soil heat flux (mm of water equivalent) due to the annual temperature cycle, from Equation 6-31.

Reflected Short Wave Radiation.

The previous discussion dealt with the soil heat flux terms in Equation 5-7:

$$\int [G_o - G_d + K_{in}(\alpha_o - \alpha_d)] dt \ll \int L_e E dt$$

Here, the possible range of values of the short wave radiation term,  $K_{in}(\alpha_o - \alpha_d)$ , will be discussed. The largest values of this term will occur immediately after irrigation when the albedo,  $\alpha_d$ , of the drying soil is much lower than that of the dry soil,  $\alpha_o$ . When the soil has reached stage III drying there will be virtually no difference in albedo and the value of the short wave radiation term will be zero. The value of albedo for a clay loam soil, similar to the one studied here but located in Phoenix, Arizona (Idso and Reginato 1974, raked soil), varied from 0.26 when dry to 0.12 when wet.

Unfortunately, the rate of change of albedo varies widely making it difficult to predict how the value of  $(\alpha_o - \alpha_d)$  in Equation 5-7 will vary with time. After irrigation in summer in a semi-arid climate the soil albedo may equal that of a dry soil within 5 days or less while during winter in the same climate it may be up to 3 weeks before the soil surface dries sufficiently so that  $\alpha_o$  equals  $\alpha_d$ . Moreover, the albedo of the drying soil may change quickly during the daylight hours varying from near wet soil values to near dry soil values in

a single day; and then, with re-wetting of the soil surface at night, albedo may again be close to that of a wet soil the next morning (Idso et al. 1974).

Not only may albedo vary with soil water content but it also varies with the solar zenith angle with albedo being highest near dawn and dusk (Monteith and Sziecz 1962, Idso et al. 1974, Aase and Idso 1975). But examination of the data of Idso et al. (1974, 1975) shows that the difference between dry and wet soil albedos remains almost constant regardless of time of day or year. For a raked Avondale clay loam at Phoenix the difference in albedos,  $\alpha_o - \alpha_w$ , is 0.14. This soil has clay and organic matter contents similar to those of the Pima clay loam used in the present study. Since particle size and organic matter content are, along with water content and surface roughness, major determinants of soil albedo (Bowers and Hanks 1965), the albedos of the Pima and Avondale clay loams should be similar.

Since surface roughness of the field soil was similar to that of the raked Avondale clay loam, the difference in albedos,  $\alpha_o - \alpha_w = 0.14$ , was assumed to be valid for the Pima clay loam. Solar radiation,  $R_s$ , data from day 92 were used since it was cloudless and calculations were performed by multiplying the half-hourly average solar radiation by 0.14 and summing the result for all positive values of  $R_s$ . The maximum value of the radiation term in Equation 5-7 was thus

found to be 1.36 mm/d for the conditions of this study.

#### Evaluation of Neglected Terms.

Both the heat flux and radiation terms in Equation 5-7 have been investigated in previous sections. Equation 5-7 is:

$$\int [G_o - G_d + K_{in}(\alpha_o - \alpha_d)] dt \ll \int L_e E dt$$

and was the first assumption used in the formulation of the energy balance model used in this study to predict evaporation from bare soil. To the degree that Equation 5-7 is untrue the energy balance model will prove inaccurate. Because the albedos were not known for intermediate days the value of the L.H.S. of Equation 5-7 could only be predicted for days immediately after irrigation, when  $(\alpha_o - \alpha_d) = 0.14$  was assumed, and for days sufficiently far from irrigation that  $(\alpha_o - \alpha_d) = 0$ . From field observations the latter condition occurred by day 99.

Table 6-7 gives the values of the individual terms in the L.H.S. of Equation 5-7 for days 92, 93, 99 and 100. The maximum value of dry soil net daily heat flux, calculated earlier, was used for all days. Net drying soil heat flux values were the average of values for ML's with closed bottoms from Table 4-11. The absolute value of the L.H.S. of Equation 5-7 varied from a high of 0.84 mm on day 99 to a low of 0.60

**Table 6-7.**

Daily integrated values of terms in L.H.S. of Equation 5-7, in equivalent mm of water evaporated, compared to average evaporation from ML's [Ave. E] in mm.

Day	$\alpha_o - \alpha_d$	Radiation Term	$G_o$	$G_d$	Heat flux Term	L.H.S.	Ave. E	% of Ave. E
92	0.14	1.36	$\leq 0.06$	0.80	$\leq -0.74$	$\leq 0.62$	3.62	17%
93	0.14	1.18	$\leq 0.06$	0.64	$\leq -0.58$	$\leq 0.60$	1.93	31%
.								
99	0.0	0.0	$\leq 0.06$	0.90	$\leq -0.84$	$\leq -0.84$	0.93	90%
100	0.0	0.0	$\leq 0.06$	0.76	$\leq -0.70$	$\leq -0.70$	0.23	304%

mm on day 93 when the radiation and heat flux terms most nearly canceled. Since the ML's were weighed late on day 92 it is probably feasible to assume that average evaporation was closer to 7 mm, the maximum value recorded and slightly less than the potential ET for that day. With that assumption the L.H.S. would be 9% rather than 17% of the R.H.S. in Equation 5-7 for the first day after irrigation. On the second day after irrigation the L.H.S. is 31% of the R.H.S. If the value of  $(\alpha_o - \alpha_d)$  were taken to be smaller on the second day, as it well might be, this percentage would be smaller.

In principle, if the values of the L.H.S. of Equation 5-7 were added to the average model estimates then the estimates should be corrected to more accurately reflect average actual evaporation. Addition of the 0.62 mm value of the L.H.S. for day 92 to the estimated evaporation of 2.70 mm (EBM3, Table

6-6) gave 3.42 mm, much closer to the measured average evaporation of 3.62 mm. Performing the same addition for day 93 gave 2.16 mm, fairly close to the actual average evaporation of 1.93 mm. For days 99 and 100 the results were not so convincing. Addition of the L.H.S. values to the average estimated evaporation for days 99 and 100 gave values of -0.11 and -0.02, respectively, when evaporation from ML's averaged 0.93 and 0.23 mm for the same days. A possible cause for this behavior is that net soil heat flux in the dry soil was higher than that calculated using Equation 6-31. This could easily be true since the reference soil was buried in the relatively cooler field and net heat flux in the field was considerably larger than that calculated by Equation 6-31 (see Table 4-11).

The value L.H.S./R.H.S. may increase for a time after irrigation as the evaporation rate decreases more rapidly than the L.H.S., but it is clear that, as the radiation term becomes smaller and the heat flux term becomes more negative, the absolute value of the L.H.S. becomes zero at some undetermined time after irrigation. After this time the radiation term goes to zero and the heat flux term becomes increasingly negative (up to a point), while the average evaporation decreases. On the 9th day after irrigation the L.H.S. was 304% of the measured evaporation. These calculations show the L.H.S. to be a much larger percentage of

daily evaporation than did the calculations of Ben-Asher et al. (1983) who found the L.H.S. to attain a maximum of 55% of daily evaporation on the 11th day after irrigation. The idea of the L.H.S. being positive immediately after irrigation, then going through zero to become negative on later days, is consistent with the behavior of EBM3 which tended to underestimate evaporation immediately after irrigation and to overestimate later on (Table 6-6).

Several conclusions seem warranted. The assumption represented by Equation 5-7 was, for the most part, not true for the conditions of this experiment. There may be some conditions for which Equation 5-7 is true. If soil warming or cooling is small and evaporative demand is large then Equation 5-7 may be more or less valid, becoming increasingly valid as the soil surface dries and albedo approaches its dry soil value. High evaporative demand is most likely to occur during summer in an arid or semi-arid climate, but irrigation in summertime is likely to cool the soil markedly with a resulting large net daily heat flux. In winter the value of  $(\alpha_o - \alpha_d)$  is likely to remain high for many days with different effects vis-a-vis Equation 5-7 depending on the concurrent magnitude of soil heat flux.

Summary.

Both energy balance models EBM2 (Equations 6-8, 6-9, 6-20 and 6-12) and EBM3 (Equations 6-8, 6-9, 6-25, 6-12 and 6-18) were significant improvements over the original EBM. This improvement resulted from three major changes in the model. Integrating with a half hour time step, instead of a 12 hour time step, allowed better interaction between temperature and wind speed effects. The introduction of a relatively easy method of accurately estimating soil surface temperature at small time intervals at many points in the field (Equations 6-8 and 6-9) resulted in much better predictions of evaporation. Likewise, the introduction of more appropriate forms of the transfer coefficient for sensible heat flux (Equations 6-25 and 6-12) improved model performance. A computer program incorporating these features is listed in Appendix G. In the next chapter estimates from both EBM2 and EBM3 will be compared to actual evaporation measured during Experiment 3.

With these changes the number of assumptions needed to formulate the model dropped from 11 to 4 resulting in a more physically based, less empirical model. The four assumptions still necessary to the model were the first four enumerated in Chapter 5 with the exception that the aerodynamic resistance of assumption 2 was traded for the transfer coefficients of Equations 6-25 and 6-12 for dry and drying soils,

respectively.

It was found that the soil heat flux and shortwave radiation terms should not be neglected (Equation 5-7, 1st assumption). Attention should be placed on methods of estimating these terms. Field estimates of albedo may be made by eye, especially if the limits of albedo are known for the soil, and could be accurate enough for first order estimates of the radiation term. But the development of a portable hand held device, similar to the infrared thermometer in concept, would be very helpful. If such a device could be incorporated into the infrared thermometer both temperature and albedo could be measured simultaneously.

The soil heat flux term in Equation 5-7 could be measured at one location and data recorded on the same device used to record weather data. If heat flux were relatively invariable in space this approach would appreciably improve the estimates of evaporation from the energy balance model. Even if only the soil heat flux term were evaluated the model would be substantially improved since the radiation term is largest when daily evaporation is largest and thus has relatively less influence on the percent error than does the soil heat flux. Measurement and/or modeling of soil heat flux in the reference is also needed to adequately describe the heat flux term.

Since the dew point was never reached in the field, negative half-hourly values of evaporation were neglected when

integrating the improved EBM's with the result that model performance improved slightly. This amounted to a shortening of the period of summation from 24 hours to about 10 hours. Underestimation of evaporation by EBM3 on the first two days after irrigation was largely explained by considering estimates of the neglected terms. Addition of estimates of the neglected terms to predicted evaporation for the last two days resulted in negative values which were most likely due to underestimation of the heat flux in the reference dry soil.

From the discussion on transfer coefficients it seems that Equations 6-25 and 6-12 did not fully describe sensible heat flux under our conditions. For instance, an attempt at stability correction of the transfer coefficient failed. However, the combined use of Equations 6-25 and 6-12 improved the predictive ability of the energy balance model greatly, with EBM3 explaining about 73 % of the variability in evaporation versus about 55 % explained by the original EBM. Experimental work should be done to accurately measure the sensible heat flux from the reference dry soil. This problem seems daunting given the need to avoid interference with insolation and air movement over the reference but perhaps eddy correlation techniques are a valid path.